# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION - **STATISTICS**

THIRD SEMESTER – NOVEMBER 2015

**ST 3506 - MATRIX AND LINEAR ALGEBRA**

Date : 06/11/2015 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

## PART - A

**Answer ALL the questions. (10 x 2 = 20 marks)**

1. Define idempotent matrix with example.
2. Mention any two properties of determinants.
3. Compute the rank of the matrix A = $\begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$.
4. Define singular and non singular matrices.
5. Define linearly dependent and independent vectors.
6. If A = $\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$ find $A^{-1}$.
7. Define a symmetric matrix.
8. Define a linear transformation.
9. Determine the characteristic roots of $\begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$.
10. Show that if $\lambda$ is the eigen value of A then $\lambda^{-1}$ is an eigen value of $A^{-1}$.

## PART - B

**Answer any FIVE questions. (5 x 8 = 40 marks)**

11. Prove that the eigen values of a hermitian matrix are all real and those of a skew hermitian matrix are purely imaginary and zero.
12. For any square matrix A with real number entries $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix.
13. Show that $(Adj\ A)A = A.(Adj\ A) = |A|I_n$ .
14. Using Cayley – Hamilton theorem, find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$.
15. Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$.
16. Show that eigen vectors associated with distinct eigen values of a matrix and linearly independent.
17. Show that a) $(AB)' = B'A'$ and b) Tr(AB) = Tr(BA).
18. Explain Cramer's rule with an example.

## PART - C

**Answer any TWO questions.** **(2 x 20 = 40 marks)**

19. a) What are the properties of determinants?

b) Show that the determinant of a matrix is left unchanged if a constant multiple of the elements of any row/column is added to the corresponding element of another row/column.

20. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 0 & 2 \\ 4 & 1 & 0 & 3 \end{bmatrix}$.

21. a) Evaluate the determinant of $\begin{vmatrix} 1 & 2 & 4 & 3 & 0 \\ 2 & 1 & -1 & 1 & 3 \\ 4 & -1 & -2 & 5 & 1 \\ 7 & 3 & 6 & 2 & 1 \\ 1 & 0 & -1 & 1 & 1 \end{vmatrix}$.

b) State and prove Cayley – Hamilton theorem.

22. a) Examine the consistency of the following system of equations and solve, if consistent:

$$x - 3y + w = 2$$

$$3y + 2z + w = 3$$

$$x - y - z - 2w = 1$$

$$-2x + 5y + z + 3w = 1$$

b) Prove the following identity.

$$\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

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